

A Teacher's Approach to Creating Learning Opportunities and Supporting Students to Make Contributions: A Case Study in Iran

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The purpose of this case study was to examine how one teacher supported her students in making contributions to whole-class discussions in an Iranian classroom. Data were analysed using a grounded theory approach and include field notes from all classroom observations and three teacher interviews collected over five weeks in a Year 11 classroom. Findings indicated that in spite of the differences in context, the Iranian teacher's practice shared substantial elements with the 'five practices' of orchestrating productive mathematical conversations outlined by Stein et al. The results suggest that proactive support of students' mathematical understandings is an achievable goal worth pursuing in Iran.

Research in mathematics education (e.g., Stein, Engle, Smith, & Hughes, 2015; Franke, Webb, Chan, Ing, Freund, & Battey, 2009) indicates that an important part of the teachers' role is to create learning opportunities for their students to explain their ideas and reason mathematically in the public space of the classroom. Research studies grounded in sociocultural perspectives provided multiple insights on how teachers create learning opportunities and support students' contributions (e.g., Franke et al., 2009; Goos, 2004). Sociocultural perspectives of learning build on the work of Vygotsky (1978) and take a position that knowledge is manifested in social and cultural practices of communities and created through joint negotiation of meaning. Students' mathematical reasoning in the classroom, where students engage in whole-class discussions, is a form of such negotiation. Within this view, the teacher's creation of opportunities for sharing ideas in the classroom can be seen as their proactive provision of support for students to participate in the negotiation of meaning, and thus in knowledge production.

In recent decades, improvement efforts in mathematics education have been pursued internationally, aimed at creating learning opportunities where students' reasoning is promoted. In Iran, the recent wave of reform efforts started in 2016, with one of the foci being to influence secondary teachers' approach to teaching mathematics. The reform has aimed at encouraging these teachers to create student-centred learning environments and to promote students' mathematical contributions. Despite these efforts, traditional models of instruction, based on transmission of meaning from an authoritative teacher to students, continue to dominate secondary mathematics teaching in Iran. Some researchers report that teachers find it challenging to support students in making their own mathematical contributions (Shahvarani & Savizi, 2007). It is thus important to provide mathematics teachers with productive images of how 'reform teaching' can look and how one could develop it in the Iranian context. Similar needs are emerging in different international contexts (Boaler, 2016), creating a need for documenting and understanding local variations of teaching practices that promote students' reasoning. The analysis reported here aimed to investigate how one teacher supported her Year 11 students in making contributions to whole-class discussions.

Literature Review

Mathematics education studies continue to draw researchers' attention to the teachers' approaches and their use of effective practices (e.g., Stein et al, 2015; Lobato, Clarke, & Burns Ellis, 2005). We first contextualise the question of effective mathematics teaching practices in Iran, and then structure the literature review around five practices highlighted by Stein et al. (2015), which we used to orient our analysis. The five practices have been reported to provide specific guidance to mathematics teachers, in the USA and beyond, in supporting students to reason mathematically in the public space of their classroom, and in creating environments in which student contributions to classroom mathematical learning are valued.

Research studies that examined secondary mathematics teacher's approaches to supporting students' contributions have been predominately conducted in 'first world' affluent countries, while limited research has been conducted in Iran. Importantly though, where Iranian studies exist, their findings on how students' learning can be supported are consistent with the international literature. For example, Atapoor and Haghverdi (2016) investigated the impact of teacher questioning on students' explanations. They randomly assigned 52 male students into control and experimental groups. For a duration of three months, the teacher prompted students' explanations in the experimental group by posing specific and probing questions (cf. Franke et al., 2009). The students in the control group were not involved in any questioning practice. The authors reported that students in the experimental group justified their ideas as they were engaged in responding to questions, which in turn improved their understanding. In contrast, the students in the control group did not improve their understanding of reasons for why their calculations worked.

Given that Iranian students benefit from opportunities to share their mathematical reasoning, it is important to explore whether and under what circumstances Iranian teachers develop teaching approaches that provide such opportunities to students. It is also important to explore how the practices of Iranian teachers relate to those documented in international research literature. We in particular follow the similarities and differences of one Iranian teacher's practice to the model of five practices (Stein et al., 2015).

Stein et al. (2015) proposed a model for teacher educators to use in their work with mathematics teachers, as they learn to support their students in making mathematical contributions in whole-class discussions. The five teaching practices in the model include: anticipation, monitoring, selection, sequencing, and connecting ideas. First, *anticipation* orients teachers to identify how students might mathematically approach the task, including how they will interpret a problem and what strategies they are likely to use to address the problem mathematically. Second, *monitoring students' responses*, orients the teacher towards students' mathematical thinking as it becomes evident while they work on the problem. The goal of monitoring is to identify strategies and ideas that are mathematically important to explore with the whole class. Third, having monitored the students' responses, the teacher then *selects* particular students to share their work with the whole class in order to get "particular pieces of mathematics on the table" (Lampert, 2001, p. 146). As Lampert (2001) summarised it, if a teacher listens to students as they are working on a problem, she is then better positioned to decide which ideas to make focal during whole-class discussions and who are the students who could share them. Fourth, the teacher also needs to make decision about the *order* in which students' ideas are to be shared. For example, Stein et al. (2015) explained that the teacher's first choice might be the strategy used by the majority of students because the teacher might want to validate their work and maximise the initial involvement in whole-class discussions. Fifth, and the last practice is to support students to *make connections* between different mathematical ideas that were being shared. This is the

phase of the lesson where students gain opportunities to learn new mathematics and explore ideas that they were not yet independently using while solving the given task. To support students' meaning making, teachers can pose questions to elicit students' responses about similarities and differences between two presented solutions (Stein et al., 2015).

Asking effective questions is critically important if students are to learn how to make connection between ideas in a whole-class situation. In a study by Franke et al. (2009), researchers identified four types of questioning: general questions, specific questions, probing sequences of specific questions, and leading questions. They characterised general questions as those related to a general issue, specific questions associated with a specific issue in students' explanations, probing sequences of specific questions related to a series of questions a student asked following several questions and responses between the teacher and the student, and leading questions associated with questions the teacher raised to orient students' explanations toward specific ideas. That is, teacher questioning often appeared as asking for clarifying explanations, uncovering the reasoning, and highlighting significant mathematical ideas. Franke et al. (2009) reported that general questions rarely supported students to provide complete explanations and the teacher's use of specific questions prompted them to elaborate on their incomplete explanations. They continued that the teacher's use of probing sequences encouraged students to clarify the strategy they used, and the leading questions supported them to give more details. The significance of examining teacher questioning relates to the idea that some questioning practices are more likely than others to provide students with opportunities to elaborate their ideas (Franke et al., 2009).

Research Design and Methods

The analysis we report here is part of a broader investigation of two contrasting approaches to teaching in Iranian high school mathematics classrooms.

Participants

The data comes from a classroom of 20 eleventh-grade female mathematics students taught by a highly-experienced teacher, Ms. Anita, during the initial five weeks of the school year. Ms. Anita was selected because, according to the first author's earlier conversations with her, she worked to align her teaching approach with the objectives of the new reform in Iran. She was selected as a contrasting case as to how mathematics is typically taught in Iran, and we hoped that documenting Ms. Anita's work would provide insights into how more Iranian teachers could start their transition to student-centred teaching.

The 20 female students in Ms. Anita's Year 11 classroom were 16-17 years old. They were taught in mixed ability groups. In prior mathematics courses, they had experienced traditional models of instruction where the teacher explained mathematical rules and they practised the procedures. They had never been encouraged to explain their ideas and share their reasoning prior to their learning with Ms. Anita.

Data Collection

The first author, who collected data as researcher-observer, first became familiar with the school in late September; and the teacher and participating students in early October. The mathematics class was held twice for 60 minutes each week. Data for this paper included field notes from classroom observations of the initial 10 lessons (5 weeks) in the school year beginning in early October. Two sets of observation field notes were collected (the second by a local mathematics educator). Immediately after each lesson, the two sets of notes were compared and combined, adding the level of detail, such as specific student and teacher quotes of particular interest. This was intended to compensate for missing classroom

recordings, which were not permitted. In addition, Ms. Anita participated in three 30 to 50 minute audio-recorded interviews in weeks 1, 3, and 5 of observations, which focused on observed classroom events. For example, the teacher was asked to explain why she encouraged students to ask questions and find their own ways of solving problems. In the last interview, Ms. Anita had reviewed the summary of her prior statements and had an opportunity to correct interpretations and elaborate on these summaries.

Data Analysis

Data analysis followed the principles of grounded theory (Strauss & Corbin, 1998). First, throughout the ongoing analysis that took place during the data collection phase, the authors documented the emerging understandings of the teacher's approach, and, when necessary, adjusted interview protocols to explore the emerging themes. In the initial phase of the retrospective analysis, the focus was on identifying when and how Ms. Anita created opportunities for students' mathematical reasoning in the public space of the classroom and how she reasoned about those decisions during her interviews.

Findings and Discussion

Here, we first present two examples from the classroom interactions to illustrate how Ms. Anita supported her students to make contributions to whole-class discussions early in the school year. We then relate these examples to Ms. Anita's views of mathematics teaching and to more summative comments that draw on the initial analysis of the entire data corpus.

The Rocket Problem

The first example comes from the third week of the school year, when one of Ms. Anita's tasks was to orient the students away from reproducing learned formulas and facts towards sense making, and use of mathematical reasoning in solving problems. This example allows us to illustrate how Ms. Anita *anticipated* students' interpretations as she planned for and used the Rocket Problem task in classroom in week 3. In the textbook, the rocket path was given as a quadratic relationship between the height h of the rocket above ground and time t that the rocket had travelled: $h(t) = 100t - 5t^2$. The task had three sub-questions asking students about: (a) the duration of rise to highest point, (b) the maximum height above ground, and (c) the duration of return to the ground. Ms. Anita noted in her interview prior to the lesson that she likes to provide students with challenging tasks, and the Rocket Problem task will likely orient students to assume that soar and fall times for a rocket flight must always be the same. She shared that she selected this task because she wanted her students to consider how the modelled behaviour might differ from a real rocket flight. She also anticipated that opportunities for such conversation would emerge when students address the sub-questions and realise that the duration of rise and fall are identical. Ms. Anita also taught physics in this Year 11 class, and was aware of tools that students could draw on from physics to elaborate and problematise the task, and deepen their thinking about how we use mathematics to model physical processes.

When this task took place in the classroom, one of the students explained that it will take the rocket 10 seconds to reach the highest point because the highest point means the maximum point of the parabola. That is,

$$\frac{-b}{2a} = \frac{-100}{-10} = 10$$

(no units were used in calculations, but seconds were mentioned in the answer). Another student explained that the height of the rocket would be zero when it returned the ground.

That is, $h(t) = 0$. Thus, $100t - 5t^2 = 0$. From this, $t_1 = 0$, $t_2 = 20$. The teacher encouraged students to explain what the two solutions meant. One of the students said, “ $t = 0$ is unrelated [to the landing time] because it should take the rocket some time to return the ground”. After the teacher nodded her head in agreement, a group of students together composed a statement suggesting that “it takes the rocket 10 seconds to reach the highest point and it takes another 10 seconds from the highest point to the ground”.

This was the moment in the conversation that Ms. Anita anticipated and planned to use for deepening students’ thinking. She asked, “Would the soar time always be equal to the return time?” Some students replied yes, while others remained silent. To encourage students to broaden how they were thinking about the scenario beyond what was given in the textbook, Ms. Anita proposed that this was not true, and asked, “Can you come up with an example where the soar time [of a rocket] could be different from the return time?” Several students provided such examples, referring to the notions of gravity and rocket velocity as additional variables (rather than parameters, as given in the textbook task). They explained that the difference between the soar and return time in their examples indicated that having data about the soar time of a rocket might not be sufficient to predict its return to the ground.

In this example, having anticipated the opportunity for this direction in students’ responses, the teacher readily provided specific questions to deepen students’ thinking about rocket flight, soar and fall times, and precision with which these could be modelled. This example was not the only occasion at which Ms. Anita anticipated students’ interpretations. Over the observed sessions, Ms. Anita chose tasks with potential to challenge students’ ideas and anticipated that certain students’ interpretations will need to be discussed and further explored. The opportunities she created for students to give examples, counterexamples, and to correct or modify their initial interpretations provide the evidence that this was inherent to how she taught mathematics.

The Square Task

We use the second example, from week five, to illustrate how the teacher intentionally *sequenced* related tasks and explanations and how she created opportunities for students to *make connections* between different facts and calculational solution methods. Ms. Anita asked the students to sketch a square and, using the textbook, dictated the Cartesian coordinates of three of its vertices— A, B, C . The task was to find the coordinates for the remaining vertex D (see Figure 1). The first student that Ms. Anita called to the board drew the diagonals of the square AC and BD , and calculated the coordinates of their intersection M as the middle point on AC , using the method that was taught in that session. The student explained, with prompts from the teacher, that each coordinate of M would be exactly half-way in between the corresponding coordinates for A and C . She continued that M is also a middle point on BD and that this can be used to calculate the coordinates of D .

Once the calculations were finished, Ms. Anita could have been satisfied with the answer and moved on to the next textbook task. Instead, she called the second student to the board, asked her to write a linear equation for AB and explain what information from past lessons was useful in doing so. The student explained that she knew that the slope of AB and coordinates of two points A and B were needed to write a linear equation. She explained how to calculate the slope and Ms. Anita encouraged all students to write the linear equation individually in their notebooks. She then asked the third student whether we could tell the slope of AD without calculations. The student responded that from last question we knew that the slope of AB is $\frac{3}{5}$. Thus, the slope of AD must be $-\frac{5}{3}$ because AB and AD met at a right angle and thus they had opposite reciprocal slopes. After that, Ms. Anita asked the

whole class to calculate the slope of AD using the same method as the second student. She asked the class to observe whether the result would be the same and verify that the inverse reciprocal rule gave the same answer. On a basic level, this would likely have reinforced the students' trust in the rule, and their awareness that other pathways are often possible when remembering the rule was difficult.

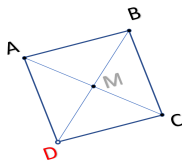


Figure 1. The square task.

Ms. Anita's use of making connections in this example was different from Stein's et al. (2015). Taken together, the reasoning used in solutions by the second and third student could have been used as an alternative pathway to calculate the 'unknown' coordinates for D , different from that presented by the first student. While the teacher did not explicitly make this connection in the classroom, it is likely that by making all the steps for such possibility available in the context of the same task, this would have made it more likely that such a solution would later emerge in the classroom without prompting from the teacher. It is possible that as the teacher continued working with the class for an extended time, the balance in the ideas that teacher needed to introduce and those that she would be able to elicit from the range of students' solutions (i.e., ideas generated by different student groups during problem solving time) would have shifted.

This example documents one of the ways how Ms. Anita supported students to think about the same problem in different ways through making careful decisions about sequencing different ideas and procedures during the whole class presentations. Even though the teacher had clear agenda about what needed to be talked about in the classroom, she consistently incorporated her students – and in particular their existing mathematical ideas and skills – in the communication space of the classroom. Ms. Anita's use of questioning practices was similar to that of Franke's et al. (2009). Instead of presenting a different solution method herself, Ms. Anita used leading questions to direct students to the particular approaches that she hoped to bring to the awareness of all students. She also used probing questions to provide opportunities for students to more fully demonstrate what they already knew and to clarify their approaches explicitly, making it thus easier for the listening students to follow the presented reasoning.

Ms. Anita's Teaching Approach

The analytical approach we used in making sense of the data guided us to combine and reconcile insights from classroom observations and from teacher interviews. Inevitably, this led us to reject some of the possible alternative interpretations for the presented classroom events and the purposes and intentions for the teacher's moves in the classroom. We thus find it important to provide a glimpse into how Ms. Anita spoke about mathematics, her teaching and the goals she had for her students.

...I'd like students to interact with each other and work in groups. Working in groups can support students to see how their peers are thinking, what their solution method is, and how they can correct their mistakes. I always acknowledge that we need to learn from each other... I'd like students to ask their questions and share their ideas because students' explanations would be an indication of their understanding...I'd like students to face with challenging tasks and find their own solution

method....I prefer the tasks that provide students with opportunities to make a link between different areas in mathematics. This can encourage students to think on the problems and [reason]....I think students need to see geometric shapes [on the board]. They need to visualise because in that way they can make sense [of mathematical problem better].

Our field notes data show that, consistent with her claims, Ms. Anita focused on promoting her students' thinking and reasoning. This became much more apparent when we contrasted the opportunities that students had for sharing their ideas in Ms. Anita's classroom with those provided by another, more traditional, teacher who took part in the larger study. As in the two examples provided here, Ms. Anita frequently asked students to think of and share examples of specific phenomena and to explain their reasoning. While some of students' explanations were calculational (cf. Thompson et al., 1994), that is, focused on how particular procedures had to be executed, in other cases, students explained procedures in ways that demonstrated how they were making sense of them (e.g., making sense visually when explaining why the x -coordinate of the midpoint M of the line segment AB must be in the middle of the x -coordinates of A and B).

Overall, the field notes data from classroom observations indicated that Ms. Anita encouraged students to work on the tasks in pairs or in groups of three students. She asked students to think aloud and verbalise their thoughts as they were engaging in their task. After allowing sufficient time for student-to-student talk, she further supported students by posing additional questions to encourage deeper considerations. Sometimes, when the questions were easier or related to recall, the teacher called on an individual student or a group to respond. Most of the time, however, all students were expected to respond to the question in their small groups and only later share their ideas in the whole class setting. As they shared, Ms. Anita often used follow-up questions to support students to explain their ideas in ways that it would become easier for the listening students to follow them. The evidence of engaging students in small group discussions and whole-class discussions, together with how the teacher facilitated student contributions, indicated to us that Ms. Anita's practices aimed at developing her students' reasoning.

Conclusion

Our initial analyses, and the examples provided in this paper, support the view that Ms. Anita was in many ways successful in supporting her Year 11 students in making contributions to whole-class discussions, even at the very beginning of the school year, while working with students who had not experienced such practices before. This is an optimistic finding, given that rich examples of classroom mathematical discussions in settings that Iranian secondary teachers recognise as relevant to their work will be needed in supporting more of them to shift how they teach mathematics in their classrooms. On the one hand, this study can serve as an existence proof that providing Iranian students with opportunities to share their ideas, make link between different mathematics areas, think about the same problem in different ways, ask questions, and correct their initial interpretations as they engage in small groups or whole-class discussions (cf. Stein et al., 2015) is possible.

On the other hand, we started to outline how analysis of the five practices and the types of teachers' questioning in our data could be used to both understand and create rich descriptions of how the teacher prepared for and orchestrated conversations in the public space of her classroom. We intend for this to lead to development of tools for supporting Iranian teachers, via orienting them to *extend the range* of the types of questions they pose in their classrooms and *consider the mathematical purposes* that can be accomplished via different forms of questioning. While the aim in such support would be the development of five practices outlined by Stein et al. (2015) for supporting students' mathematical reasoning, questioning appears to provide a rather approachable avenue for explorations of

own practice also for secondary mathematics teachers (cf. Franke et al., 2009). We would like to explore those possibilities in Iranian context.

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